Interferometric angular velocity measurement of rotating blades: theoretical analysis, modeling and simulation study

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Abstract: Doppler radar can only measure the radial velocity of a moving object. If an object is moving along a curved path, when its radial velocity decreases, the angular velocity must increase. Thus, if the angular velocity can be measured, the problem caused by little or no radial velocity can be solved. In this paper, we provide detailed theoretical analysis and establish the mathematical model of the interferometric frequency shifts of rotating blades. We first analyze the instantaneous frequency of a SINC function, which comprises a pair of sinusoidals and a train of strong spectrum lines. Then, we utilize the convolution theory in time-frequency domain to calculate the interferometric frequency shifts of rotating blades. Simulation results manifest that some important parameters and features of rotating blades, such as blade length, rotating rate and blade number, can be accurately estimated from the time-varying interferometric frequency signatures.

1 Introduction

Consider a radar observing an object moving along a curved path in the far field as shown in Fig. 1. The distance between the radar and the object is denoted as $R$ and the incident angle of the object is denoted as $\phi$ relative to the antenna broadside. The linear velocity of the moving object is a vector, with its magnitude measuring the rate of change of its position displacement, and its direction along the tangential of the trajectory, as represented by the velocity $v$ in Fig. 1. The velocity $v$ can be decomposed into two orthogonal components with reference to the radar, those are radial velocity $v_r = v \cos \alpha$ along the direction of the radar line of sight (LoS) and the cross-radial velocity $v_{cr} = v \sin \alpha$ perpendicular to the radar LoS, with $\alpha$ defined as the angle between the radar LoS and the object linear velocity. The radial velocity causes the frequency of received signals shifted from the carrier frequency, which is referred to as Doppler frequency [1, 2]. Clearly, when the trajectory of the object approaches an angular trajectory, i.e. $\alpha \to 90^\circ$, the radial velocity relative to the radar is decreasing, while the cross-radial velocity increases. Beyond $\sim 60^\circ$, studies have shown that the Doppler frequency shift induced by radial velocity is reduced significantly enough to prevent sufficient features for classification [3, 4].

The cross-radial velocity contributes to the angular displacement of the object relative to the antenna broadside and the change rate is characterized by angular velocity, $\omega$, defined by revolutions per second (rps) or radians per second (rad/s). The angular velocity $\omega$ of an object is given by its change rate in angle over time and equal to the cross-radial velocity $v_{cr}$ divided by the range $R$ between the object and the radar,

$$\omega = \frac{\partial \phi}{\partial t} = \frac{v_{cr}}{R}. \quad (1)$$

where $\phi$ represents the arrival angle of the object relative to radar. As such, the frequency shift induced by angular velocity is inversely proportional to the range, whereas the Doppler frequency shift induced by radial velocity is independent of range.

The Doppler radar is well-known for measuring the radial velocity of a moving object, however, it is unable to measure the angular velocity of the object. The angular velocity can be measured by an interferometric radar, which correlates the outputs of two widely spaced receiving antennas [5–8]. The correlation interferometer has been used in radio astronomy for the remote sensing of the earth [9]. An object passing through the interferometric beam pattern produces an oscillation in the signal output whose frequency is directly proportional to the angular velocity of the object. When the depression angle of rotating blades is $90^\circ$ relative to the radar, radial velocity observed by the radar diminishes completely, while angular velocity remains visible. The interferometric radar is capable of providing the complete micro-Doppler signatures of an arbitrary moving object regardless of the trajectory, thus significantly improving the capability of radar target detection and classification.

The phenomenon of micro-Doppler in radar was introduced in [10–12] to characterize the micro-motion dynamics of moving objects. These micro-motions modulate the Doppler frequency induced by the target bulk motion, which are termed as the micro-Doppler frequency [10–13]. Utilizing the time-varying micro-Doppler spectrum, the micro-motion information of some basic structures can be extracted and analyzed [14, 15], such as aircrafts [2], unmanned aerial vehicles (UAVs) [16–19], human being [20–24] and animals [25, 26] to list a few. The mathematical model of micro-Doppler modulations induced by radial velocity was derived by Chen in [10–12]. The interferometric radar was utilized in [5–8,
Finally, we validate the theoretical analysis with spectrogram of the blades, thus paving the road for further potential research on between the rotating center and the second receiving antenna is shown in Fig. 2. The interferometric radar is located at the origin of the coordinates with a translation of \( H = (0,0,0) \) along the \( Z \)-axis. The distance \( \psi_n \) of moving targets, this work can enhance the performance of target detection and classification based on micro-Doppler signatures only. Finally, we validate the theoretical analysis with spectrogram obtained from short-time Fourier transform (STFT) using simulation and experimental data.

The rest of the paper is organized as follows. The mathematical model of interferometric radar and rotating blades is introduced in Section 2. Section 3 conducts the mathematical analysis of the interferometric micro-Doppler spectrum induced by angular velocity. Sections 4 and 5 provide the simulation and experimental results to validate the correctness of proposed theoretical analysis. Finally, the conclusion remarks are given in Section 6.

2 Mathematical model of interferometric radar and rotating blades

2.1 Geometry of interferometric radar and rotating blades

The geometry of the interferometric radar and rotating blades is shown in Fig. 2. The interferometric radar is located at the origin of the space-fixed coordinates \((X, Y, Z) = (0,0,0)\). The two receiving antennas locate on the \( Y \)-axis with positions \((0,0,0)\) and \((0,D,0)\), respectively. The rotor blades locate above the radar and are centered at the origin of the body-fixed coordinates \((x, y, z) = H_b\) rotating about the \( z \)-axis with an angular rotation rate \( \Omega \) rad/s. That is, the body-fixed coordinates are parallel with the space-fixed coordinates with a translation of \( H_b \) along the \( Z \)-axis. The distance between the rotating center and the second receiving antenna is denoted as \( R_e = \sqrt{H_e^2 + D^2} \). Under the described geometry, the arrival angle of the rotor is broadside relative to the first receiving antenna and its observed radial velocity is zero.

From the electromagnetic scattering point of view, each blade of the rotor consists of scatterer centers. Each scatterer center is considered as a point with a certain reflectivity. For simplicity, the same reflectivity is assigned to all the scatterer centers [11]. If the initial rotation angle of the scatter point \( P \) of the \( n \)th blade at \( t = 0 \) is \( \psi_n \), then at time \( t \) the rotation angle becomes \( \psi_n + \Omega t \). We define the time duration \( T \) such that \( \Omega T = \pi \), that means \( T \) is the half rotation cycle. For a rotor with \( N \) blades, the \( N \) different initial rotation angles with the following relationship,

\[
\psi_n = \psi_0 + \frac{2\pi(n-1)}{N}, \quad n = 1, \ldots, N.
\]

where \( \psi_0 = \psi_0 \) denotes the initial angle of the first blade.

2.2 Basic principles of interferometric radar

An interferometric radar consists of one transmitting antenna and two nominally identical receiving antennas spaced by a baseline \( D \) observing a far-field monochromatic point source \( s(t) \), as shown in Fig. 3. Assume that the source signal is impinging on the interferometric radar from the angle \( \phi \) measured from the broadside direction. Taking the signal received by the first antenna as a reference, that is,

\[
s_1(t) = A(\phi) \exp \left( -j2\pi f_s t \right).
\]

The geometrical time delay \( \tau \) of reception of the signal between the two antennas can be expressed as,

\[
\tau = \frac{D \sin \phi}{c},
\]

where \( c \) denotes the speed of light. The two received signals are correlated in a device, called a correlation interferometer, generating the interferometer response,

\[
c(t) = s_1(t) \times s_2(t) = \left| A(\phi) \right|^2 \exp \left( -j2\pi f_s \tau \right).\]

Taking the time derivative of the phase term of the interferometer response in (6), the fringe frequency is therefore,

\[
f_f = \frac{D \cos \phi}{\lambda} \frac{\partial \phi}{\partial t} = \frac{D \cos \phi}{\lambda} \omega,
\]

where \( \omega \) denotes the angular velocity of the moving object. Note that the angular velocity \( \omega \) is different from the angular rotation rate \( \Omega \), where the former is measured relative to the observing radar and the latter is referred to the body-fixed coordinates. Equation (7) indicates that the fringe frequency detected by the interferometer will decrease as the object moves away from broadside in either direction. This can be seen in Fig. 3, where the in-phase part of the correlator output \( c(t) \) is plotted. The center frequency of the radar is set as \( f_c = 24 \text{ GHz} \) and the baseline \( D = 10 \lambda \) with \( \lambda = c/f_c = 1.25 \text{ cm} \) denoting the wavelength. Clearly, the correlator response varies sinusoidally as the source moving across the space, i.e. as \( \phi \) changes from \(-90^\circ\) to \(90^\circ\). This sinusoidal output is referred to as the interferometer ‘fringe pattern’ in radio astronomy literature [9], which can be utilized for radio astronomical imaging. It can be observed that the fringe frequency slows as the angle off broadside of the object increases.
If the incident radiation is not monochromatic, the effect of bandwidth on the correlation signal is simply the integration over the bandwidth \( \Delta f \) of the signal or the receiver, depending on which bandwidth is narrower [3–5, 7]. For an ideal square passband, the signal output in (6) is integrated over the bandwidth \( \Delta f \) around the carrier frequency \( f_c \), giving that
\[
\tilde{c}(t) = A(\theta) \left| \int_{f_c - \Delta f}^{f_c + \Delta f} \exp\left(-j2\pi f \frac{D\sin{\phi(t)}}{c}\right) df, \right|
\]
\[
= A(\theta) \left| \int_{f_c - \Delta f}^{f_c + \Delta f} \exp\left(-j2\pi f \frac{D\sin{\phi(t)}}{c}\right) \sin{\left(\pi \Delta f \frac{D\sin{\phi(t)}}{c}\right)} \right|. \tag{8}
\]
The sinc function in (8) is called the bandwidth pattern and has the effect of angularly limiting the interferometer response as the antenna pattern \( A(\theta) \). For the remainder of this paper, we assume the radar transmit signal is narrowband and the receiving antennas are isotropic, thus both bandwidth and antenna pattern can be neglected from the correlation output.

3 Interferometric frequency shift induced by angular velocity

We provide a mathematical model of interferometric frequency shift induced by angular velocity of rotating blades in this section. As shown in Fig. 2, the rotor center is placed above the first receiving antenna, i.e. the depression angle of the rotor is \( \psi = 90^\circ \). By neglecting the interference of the first receiving antenna. Considering a scatter point \( P \) on the \( n \)th blade, its position \( P(t) \) in the space-fixed coordinates at time instant \( t \) is,
\[
P(t) = \left[ l_p \cos(\psi_n + \Omega t), l_p \sin(\psi_n + \Omega t), H_n \right]^T, \tag{9}
\]
where \( l_p \) denotes the distance between the scatterer point \( P \) and the origin of the body-fixed coordinates and \( H_n \) is the height of rotating rotor. According to (6), the correlation radiometer output can be expressed as,
\[
\tilde{c}(t) = C \left| \int_{f_c - \Delta f}^{f_c + \Delta f} \exp\left(-j2\pi f \frac{D\sin{\Omega t + \psi_n}}{2\lambda R_0}\right) df, \right|
\]
\[
= C \left| \int_{f_c - \Delta f}^{f_c + \Delta f} \exp\left(-j2\pi f \frac{D \sin{(\Omega t + \psi_n)}}{2\lambda R_0}\right) \right|, \tag{10}
\]
where \( C = |A| \exp\{ -j2\pi (D^2/2\lambda R_0) \}. \) In the second line of (10), we approximate the sine of the arrival angle with \( (D - l_p \sin(\Omega t + \psi_n))/R_0 \). Integrating the correlation output over the entire blade length yields,
\[
\tilde{c}(t) = C \int_{0}^{L} \tilde{c}(t)dl_p,
\]
\[
= CL \int_{0}^{L} \exp\left(-j2\pi f \frac{D \sin{(\Omega t + \psi_n)}}{2\lambda R_0}\right) dl_p, \tag{11}
\]
where
\[
\Phi_\delta(t) = B \sin{(\Omega t + \psi_n)} = \frac{DL}{2\lambda R_0} \sin{(\Omega t + \psi_n)}. \tag{12}
\]
We refer the sinc function in (11) as to the baseline function. For a rotor with \( N \) blades, the total correlation output can be written as,
\[
\tilde{c}(t) = CL \sum_{n=1}^{N} \exp\{ j2\pi \Phi_\delta(t) \} \sin\{ 2\pi \Phi_\delta(t) \}. \tag{13}
\]

3.1 Interferometric frequency of a rotor with one blade

First, assume a rotor with one blade for the purpose of mathematical analysis. Without loss of generality, we set the initial angle to \( \psi_i = 0 \). The correlation interferometer output can be simplified as,
\[
\tilde{c}(t) = CL \exp\{ j2\pi B \sin(\Omega t) \} \sin\{ 2\pi B \sin(\Omega t) \}. \tag{14}
\]
It is worth noting that the sinc function also produces a time-varying spectrum and cannot be viewed as a pure amplitude. The instantaneous frequency of a sinc function comprises three components: a pair of sinusoids and a train of spectrum lines located at time instants \( t = kT \), \( k \in \mathbb{Z} \), spanning the frequency range \( [-f_m, f_m] \), where \( f_m = B_2 = D\Omega/2\lambda R_0 \) and \( \mathbb{Z} \) denotes the set of non-negative integers. That is,
\[
\mathcal{F}\{ \sin\{2\pi B \sin(\Omega t)\} \} = \{ \pm f_m \cos(\Omega t), R(-f_m, f_m, kT) \}. \tag{15}
\]
where \( R(-f_m, f_m, kT) \) denotes a rectangular spectrum with uniform intensity spanning the frequency range \( [-f_m, f_m] \) at time instants \( kT \). The symbol \( \mathcal{F}\{ h(t) \} \) represents the instantaneous frequency of the process \( h(t) \). From the expression of the maximum frequency shift \( f_m \), we can see that the maximum frequency shift induced by the angular velocity decreases with the range \( R_0 \) and increases with the baseline \( D \).

Proof: Proceeding from (14), the sinc function is defined as,
\[
h(t) = \sin\{2\pi \Phi(t)\} = \sin\{2\pi B \sin(\Omega t)\}. \tag{16}
\]
According to the definition of sinc function, \( h(t) \) can be rewritten as,
\[
h(t) = \exp\{ j2\pi \Phi(t) \} - \exp\{ -j2\pi \Phi(t) \}\]
\[
= \frac{\cos(2\pi f_m t)}{2\pi}. \tag{17}
\]
Note that the sinc function \( h(t) \) exhibits an abrupt change at time instants \( t = kT \), \( k \in \mathbb{Z} \), and otherwise changes smoothly. The instantaneous frequency of \( h(t) \) at time \( t \neq kT \) can be obtained by taking time-derivative of two phase terms in (17), which comprises two components,
\[
\mathcal{F}\{ h(t) \}_t |_{t \neq kT} = \{ \pm f_m \cos(\Omega t) \}. \tag{18}
\]
Next, consider the instantaneous frequency of \( h(t) \) at time instants \( t = kT \). The phase term \( \Phi(t) \) at time instants \( t = kT \) can be rewritten as,
\[
\Phi(t)_t |_{t = kT} = \Phi(kT) + \Phi'(kT)(t - kT),
\]
\[
= -f_m \cos(kT)(t - kT),
\]
\[
= (-1)^k f_m (t - kT), \tag{19}
\]
where \( \Phi(kT) = 0 \) and \( \Phi'(t) = -f_m \cos\Omega t \) is the first-order derivative of \( \Phi(t) \) with respect to \( t \). Utilizing (19), the sinc function at time instants \( t \neq kT \) can be rewritten as,
\[
h(t) = \sin\{(-1)^k \cos(2\pi f_m t - kT)\} \delta(t - kT),
\]
\[
= \sin\{2\pi f_m (t - kT)\} \delta(t - kT). \tag{20}
\]
where \( \delta(t - t_0) \) denotes a Dirac Delta function shifted to \( t_0 \). The instantaneous spectrum of \( \sin\{2\pi f_m (t - kT)\} \) is a rectangular spectrum at time instants \( t = kT \), i.e.
\[
R(-f_m, f_m, kT) = u(f + f_m) - u(f - f_m), \tag{21}
\]
where \( u(f) \) denotes the unit step function,
with a height of is calculated as

Substituting them into (13), yields the correlation interferometer

The instantaneous frequency of the exponential function at any

where denotes the convolution operation. Thus, the

Assume that the interferometric radar is operating at 24 GHz

maximum frequency and a train of spectrum lines located at the time instants \( t = kT \), \( k \in \mathbb{Z} \), spanning the frequency range \([-2f_{\text{aw}}, 2f_{\text{aw}}]\). That is,

Utilizing the same parameters as in Fig. 4, the interferometric

As explained in Section 3.1, the instantaneous frequency of the function \( \exp{j2\pi \Phi(t)} \) sinc\{2\pi \Phi(t)\} consists of four components, which is copied here,
divide each blade into 120 facets as shown in Fig. 7. The arrival is assumed to be the geometric centroid of its triangle vertices. We presented in Fig. 6 utilizing the same parameters as in Fig. 4. The wavelength of arrays of triangular facets. The scatterer center of each triangle is located at instant times \( t = (k/3)T, k \in \mathbb{Z} \), spanning the frequency range \([-882 \text{ Hz}, 882 \text{ Hz}]\). Moreover, the maximum frequency is 882 Hz. A train of alternative spectrum is (0, 3.75, 10)m. Based on the Physical optics (PO) facet radar cross section model, a rectangular blade can be represented by the arrays of triangular facets. The scatterer center of each triangle is assumed to be the geometric centroid of its triangle vertices. We divide each blade into 120 facets as shown in Fig. 7. The arrival angle of each facet can be calculated from the radar location and the position of the geometric centroid of the facets. The interferometric spectrogram is presented in Fig. 8. We can clearly observe that the spectrogram comprises a train of spectrum lines spanning the frequency range \([-882 \text{ Hz}, 882 \text{ Hz}]\). Moreover, the interval between the adjacent spectrum lines is \( T = 0.1 \text{s} \), from which we can calculate the rotation rate of the blade is

\[
\mathcal{F}\{\exp[j2\Phi(t)]\text{sinc}(2\Phi(t))\} = \{0, 2f_m\cos(\Omega t), R(0, 2f_m, 2kT), R(-2f_m, 0, (2k+1)T)\}.
\]  
(29)

Conducting a similar analysis, the instantaneous frequencies of the other two terms in (28) are

\[
\mathcal{F}\{\exp[j2\Phi_1(t)]\text{sinc}(2\Phi_1(t))\} = \{0, 2f_m\cos\left(\Omega + \frac{2\pi}{3}\right), R\left(-2f_m, 0, 2kT + \frac{T}{3}\right)\}
\]

\[
R\left(0, 2f_m, 2kT + \frac{4T}{3}\right)\},
\]

and

\[
\mathcal{F}\{\exp[j2\Phi_2(t)]\text{sinc}(2\Phi_2(t))\} = \{0, 2f_m\cos\left(\Omega + \frac{4\pi}{3}\right), R\left(0, 2f_m, 2kT + \frac{2T}{3}\right)\}
\]

\[
R\left(-2f_m, 0, 2kT + \frac{5T}{3}\right)\}
\]

respectively. The interferometric frequency of a three-blade rotor can be obtained by combining (29)–(31), which includes a DC component, a triple of sinusoidals with different initial phases and two trains of spectrum lines spanning the respective frequency ranges \([-2f_m, 0]\) and \([0, 2f_m]\), i.e.

\[
\begin{align*}
0, 2f_m\cos\left(\Omega + \frac{(n-1)2\pi}{3}\right), & R(0, 2f_m, 2kT), \\
R\left(-2f_m, 0, \frac{2k+1}{3}T\right)\}, & n = 1, 2, 3; k \in \mathbb{Z}.
\end{align*}
\]

(32)

It is worth noting that although interferometric frequency as shown in (27) and (32) exhibits similar shape with micro-Doppler frequency derived in [10, 11], the two frequencies are essentially different. The interferometric frequency is generated when objects move across the fringe pattern of a correlation interferometer and proportional to the objects' cross-radial velocity, while the micro-Doppler frequency is caused by the radial velocity of moving objects relative to the radar. The interferometric frequency of an arbitrary \(N\)-blade rotor can be derived in a similar way.

The interferometric spectrogram of a three-blade rotor is presented in Fig. 6 utilizing the same parameters as in Fig. 4. The maximum frequency is 882 Hz. A train of alternative spectrum lines located at instant times \( t = (k/3)T, k \in \mathbb{Z} \), spanning the frequency range \([0 \text{ Hz}, 882 \text{ Hz}]\) for even \(k\) and \([-882 \text{ Hz}, 0 \text{ Hz}]\) for odd \(k\).

4 Simulations

First, consider the interferometric frequency of a two-blade rotor. Assume that the first antenna of the 24 GHz interferometric radar is located at the space-fixed coordinates \((0, 0, 10)m\) with a maximum frequency shift of 882 Hz. Based on the interferometric spectrogram of a simulated two-blade rotor using PO facet model

\[
\Omega = 2\pi/(2T) = 10\pi\text{rad/s},
\]

which coincides with the true value. From the formula of maximum frequency shift, the blade length is calculated as \(L = (\pi R f_m)/(D\Omega) = 1\text{ m}\), which agrees with the true value as well.

Next, we examine the interferometric spectrogram of a three-blade rotor. The parameters are set as the same as Fig. 8. Based on the PO facet radar cross section model, we divide each blade into 120 facets as shown in Fig. 9. The interferometric spectrogram of the three-blade rotor is presented in Fig. 10. We can observe that a train of alternative spectrum lines located at time instants \( t = (k/3)T, k \in \mathbb{Z} \), spanning the frequency range \([-882 \text{ Hz}, 882 \text{ Hz}]\) for even \(k\) and \([-882 \text{ Hz}, 0 \text{ Hz}]\) for odd \(k\).

5 Experiments

The experimental setup is shown in Fig. 11. A two-blade rotor is placed in front of the first receiving antenna of the interferometric
The blade length is $L = 0.15$ m. The rotor is facing toward the radar and rotating with a constant velocity of $\Omega = 6\pi$ rad/s. The radar is transmitting a continuous-time waveform with a frequency of 24 GHz and a wavelength of $\lambda = 1.25$ cm. The baseline of the two receiving antennas is set as $D = 0.6$ m. The Doppler spectrogram of the first receiving antenna and the interferometric spectrogram are presented in Figs. 12 and 13, respectively. As the rotating blade does not exhibit radial velocity relative to the radar, the spectrogram of the first receiving antenna consists of DC components only without Doppler frequency shift, as shown in Fig. 12. However, we can clearly observe that the interferometric spectrogram is periodic and the interval between the adjacent spectrum lines is $\sim 0.17$ s, as shown in Fig. 13. Thus, we can deduce that the rotation rate of the blades is $2\pi/0.34 = 5.89\pi$ rad/s, which is close to the true value of $6\pi$ rad/s. We can also see that the maximum interferometric frequency shift is 131 Hz in average, which agrees with the true value of $f_m = 126$ Hz as well. In a nutshell, the experimental results validate the theoretical analysis proposed in this paper.

6 Conclusions

The interferometric micro-Doppler spectrum induced by angular velocity of rotating blades in time-frequency domain was theoretically modelled and mathematically derived in this paper. When the depression angle of rotating blades was set as 90° relative to the radar, radial velocity observed by the Doppler radar diminished completely, while angular velocity measured by interferometric radar exhibited the maximum value. The Doppler...
frequency shift induced by radial velocity failed to provide sufficient information for target classification. Whereas some basic parameters of rotating blades, such as blade length, rotation rate and blade number can be deduced from the interferometric micro-Doppler spectrum, thus providing complementary micro-motion information. Both simulated radar data and experimental data measured by an interferometric radar verified correctness of the theoretical analysis proposed in this work. The investigation of interferometric angular velocity in the case of an arbitrary depression angle will become a future research topic.

7 References